A New Response Model for Multiple-Choice Items

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Abstract

Approaches for modeling responses to multiple-choice items fall into two broad categories: (a) those that group all distractor options into a single incorrect response category and model the probability of correct response using a dichotomous response model, and (b) those that retain the distinction between all response options and model the probability of each response option using a polytomous response model. A recently developed polytomous model for multiple-choice items is Revuelta's (2005) generalized distractor rejection model (DLT). A drawback of the DLT is that it is parameterized using a form that is inconsistent with many widely used response models in educational measurement. In this paper we propose an adaptation of the DLT called the distractor model (DM). The DM uses a parameterization that is consistent with that of other widely used response models in educational measurement, and thus may be more accessible and intuitive for applied test developers than the original form of the DLT. We present the derivation of the DM, its relationship to the parameterization of the DLT, and describe the relationship of the DM to other polytomous response models for multiple-choice items. In addition, we illustrate the use of the DM by applying it to responses to an algebra test composed of multiple-choice items.

Index terms: item response theory, polytomous models, multiple-choice items

A New Response Model for Multiple-Choice Items

Multiple-choice items are a widely used item format in tests of achievement, knowledge, and ability (Osterlind, 1998). The multiple-choice item format has the defining property of having one option (i.e., the key) designated as the correct response, and all other options designated as distractors or incorrect responses. Numerous response models for multiple-choice item formats have been proposed, and these models can be grouped into two broad categories: dichotomous and polytomous. Dichotomous models treat the item response as a binary variable by collapsing all distractors into a single category of incorrect response, and specifying the probability of correct response as a function of the latent trait. Dichotomous response models include, but are not limited to, the Rasch model (Rasch, 1960) and the one-, two-, and three-parameter logistic models (Birnbaum, 1968; Lord, 1980). In contrast to dichotomous models, polytomous response models retain the distinction between all response options, and specify the probability of each possible response option as a function of the latent trait.

The advantage of using a polytomous model for multiple-choice items stems from the potential of extracting information from the distractors as well as the correct response. The incorporation of information pertaining to each of the distractors maximizes the information concerning the latent trait, and thus has the potential to lead to more precise estimation of the latent trait than the dichotomous response models, particularly at the lower end of the latent trait continuum (Bock, 1972; De Ayala, 1989, 1992; Thissen, 1976). Polytomous models, however, contain a higher number of parameters than their dichotomous counterparts, and thus require greater sample sizes to be effectively implemented (De Ayala & Sava-Bolesta, 1999; DeMars, 2003).

Several polytomous response models for multiple-choice items have been proposed. The most widely used of these models is the nominal response model (NRM; Bock, 1972). To describe the NRM, let us consider a multiple-choice item with m outcomes, of which $m - 1$ are distractor options. Let the response options be denoted by k, where $k = 1, \ldots, m$. Let us denote the response variable for the jth item by Y_j, such that a response equal to the kth option is denoted by the outcome $Y_j = k$. Based on this formulization, the NRM specifies the probability of observing response option k of the jth item conditional on target trait, θ, by

$$P(Y_j = k \mid \theta) = \frac{\exp(b_{jk} + a_{jk}\theta)}{\sum_{k'=1}^{m} \exp(b_{jk'} + a_{jk'}\theta)} . \qquad (1)$$

In Equation 1, b_{jk} is a location parameter associated with the kth response option and a_{jk} is the slope parameter associated with the kth response option. In applying the NRM to multiple-choice items, the option with the largest positive value of a_{jk} will be monotonically increasing, and thus this option corresponds to the correct option. In order for the model to be identifiable, the following constraints are imposed: $\sum b_{jk} = 0$ and $\sum a_{jk} = 0$. As a result, the NRM has $2(m-1)$ free parameters.

To aid in the description of the model, let us assume that option $k = m$ is reserved for the correct option, and $k = 1, 2, \ldots, m-1$ corresponds to the remaining options (i.e., the distractors). Thus, $Y_j = m$ corresponds to a correct response. It is relevant to note that the NRM form presented in Equation 1 is based on the assumption that an individual presented with the choice between the correct option and the kth distractor will select the kth distractor with probability

$$P(Y_j = k \mid \theta, Y_j = k, m) = \frac{\exp(b^*_{jk} + a^*_{jk}\theta)}{1 + \exp(b^*_{jk} + a^*_{jk}\theta)} \tag{2}$$

for $k \neq m$, and $a^*_{jk} = a_k - a_m$ and $b^*_{jk} = b_k - b_m$. A proof of the form shown in Equation 2 is provided in the Appendix. The form shown in Equation 2 will be referred to here as the *k*th contrast function, and is equivalent to the two-parameter logistic function commonly employed for dichotomously scored items (Lord, 1980). That is, the probability of selecting the *k*th distractor given the choice between the correct option and the *k*th distractor is assumed to follow the two-parameter logistic model shown in Equation 2. This model has lower and upper asymptotes of 0 and 1, which has important implications for the fit of the NRM to real multiple-choice response data (discussed below) and for the development of the DM as an alternative to the NRM for modeling multiple-choice item responses.

While the NRM provides a flexible mechanism for modeling response data, it has a theoretical limitation in its application to multiple-response items. Under the NRM, the option having the most negative a_{jk} is modeled with a trace line that is monotonically decreasing, having lower and upper asymptotes of 0 and 1. This poses a problem because it assumes that a respondent with an arbitrarily low level of θ will have a near zero probability of selecting any option other than the option with the most negative a_{ij}. This contradicts our intuitive notion of guessing, whereby an individual with an arbitrarily low level of θ would guess at the correct response, and thus could have a meaningfully non-zero probability of selecting any of the response options. As a result, the NRM may experience poor fit to the responses of some items, particularly at the lower end of the θ continuum. We acknowledge that while it is possible for the NRM to fit actual multiple-choice response data well within a specific range of θ (depending on the specific parameter values of the NRM), the NRM itself provides no inherent mechanism to model guessing at the lower range of θ.

To address the limitations of the NRM to model guessing in the multiple-choice item responses, a multiple-choice model (MCM) was proposed by Thissen and Steinberg (1984). The MCM is an extension of the NRM in which a latent "don't know" category is included in the model. As such, the MCM contains m + 1 response categories, denoted by k = 0, 1, …, m, whereby category k = 0 corresponds to the "don't know" category. The probability belonging to the "don't know" category given a selection of the kth observed score category is captured by a separate parameter, d_{jk}. The conditional probability of selecting the kth response option is given by

$$P(Y_j = k \mid \theta) = \frac{\exp(b_{jk} + a_{jk}\theta) + d_{jk}\exp(b_{j0} + a_{j0}\theta)}{\sum_{k'=0}^{m} \exp(b_{jk'} + a_{jk'}\theta)}. \tag{3}$$

Because the d_{jk} parameters represent a set of proportions, they have the constraint $\sum d_{jk}$ = 1. Unlike the NRM model, which contains 2(m – 1) free parameters, this version of the MCM has 3m – 1 free parameters. A restricted version of the MCM was proposed by Samejima (1979) in which d_{jk} was set equal to the constant value of 1/m, representing the situation of equal guessing across the observed response options. Samejima's restricted version of the MCM contains 2m free parameters.

Recently, Revuleta (2005) proposed an alternative polytomous response model for multiple-choice items called the generalized distractor rejection model (DLT). The DLT is based on a contrast function that differs from that of the NRM (see Equation 2). Rather than defining the contrast function using $P(Y_j = k|Y_j = k, m) = P(Y_j = k)/[\, P(Y_j = k) + P(Y_j =$

m)], the DLT is predicated on a contrast function of $P(Y_j = m)/P(Y_j = k)$, which corresponds to the odds of selecting the correct option given the choice between the correct option and the kth distractor. In the description of the DLT, this contrast function is referred to as the distractor selection ratio. Note that the kth contrast function of the NRM describes the probability of selecting the kth distractor given the option between the kth distractor and the correct response, and the kth distractor selection ratio describes the odds of selecting the correct response given the option between the kth distractor and the correct response. The kth distractor selection ratio is modeled using

$$\psi_{jk}(\theta) = \frac{P(Y_j = m \mid \theta)}{P(Y_j = k \mid \theta)} = \frac{\pi_{jm}}{\pi_{jk}}\left[1 + \exp(\beta_{jk} + \alpha_{jk}\theta)\right], \tag{4}$$

where b_{jk} corresponds to the location of the jth distractor, a_{jk} corresponds to the discrimination of the kth distractor, and π_{jk} corresponds to the probability of selecting the kth response option as $\theta \to -\infty$ (i.e., the parameter π_{jk} reflects the probability of selecting the kth response option for an individual with an arbitrarily low level of θ). Using this parameterization for the kth distractor selection ratio, the DLT posits the probability of selecting the kth response option using

$$P(Y_j = k \mid \theta) = \frac{\omega_k}{\sum_{k'=1}^{m-1} \omega_{k'}}, \tag{5}$$

where

$$\omega_k = \begin{cases} 1, & \text{if } k = m \\ 1/\psi_k(\theta), & \text{if } k \neq m \end{cases}$$

The details of the derivation of the DLT can be found in Revuelta (2005).

Two noteworthy properties of the DLT are: (a) the number of free parameters to be estimated for each item is equal to $3(m - 1)$, which is fewer than that of Thissen and Steinberg's (1984) MCM; and (b) the value of π_{jk} can be interpreted directly with respect to the guessing attractiveness of each response option (i.e., π_{jk} equals the probability of the kth response option being selected by an individual with an arbitrarily low level of θ).

It is relevant to note, however, that the parameterizations of the DLT and NRM are based on different models. The parameterization of the NRM is founded on modeling the probability of selecting the kth distractor given the choice between the kth distractor and the correct response using the contrast function described in Equation 2. In contrast, the parameterization of the DLT is founded on modeling the odds of selecting the correct response versus the kth distractor using the distractor selection ratio described in Equation 4. As a result, the NRM and DLTM hold different interpretations of the location (b_{jk} and β_{jk}) and discrimination (a_{jk} and α_{jk}) parameters. Given the widespread use of the NRM, the inconsistency of the DLT's parameterization with that of the NRM poses a potential obstacle to the interpretation and utilization of the DLT. In addition, the DLT defines α_{jk}, β_{jk}, and π_{jk} according to different functions; α_{jk} and β_{jk} are defined according to the distractor selection ratio model while π_{jk} is defined according to the option response functions of the DLT. That is, α_{jk} and β_{jk} are interpreted with respect to the odds of selecting the correct response given the choice between the correct response and the distractor in question, while π_{jk} is interpreted with respect to the probability of selecting the particular response options for an individual with an arbitrarily low value of θ. As a result of these drawbacks, interpretation of the DLT parameters is hampered, particularly with respect to understanding the relationship between the parameters of the DLT and those of the NRM.

In this paper we propose an alternative parameterization of the DLT that is consistent with widely used item response models. The alternative parameterization will be referred to as the distractor model (DM), in an effort to distinguish it from the parameterization used in the DLT. Because the parameterization of the DM is consistent with widely used response models, it may facilitate the use and understanding of the model relative to the parameterization of the DLT proposed by Revuelta (2005). In this paper we present the parameterization of the DM and an application of the DM to a real data set and provide concluding remarks.

The Distractor Model

The development of the DM begins by establishing an explicit model for $P(Y_j = k|\theta, Y_j = k, m)$, similar to the development of the NRM (see Equation 2). Note that this contrasts the development of the DLT, which is based on parameterizing the odds of $Y_j = k$ given the choice between options k and m (see Equation 4). The DM is based on the assumption that the probability of selecting the kth distractor, given that the response is either the kth distractor ($Y_j = k,\ k \neq m$) or the correct option ($Y_j = m$), can be modeled using the parametric form

$$P(Y_j = k \mid \theta, Y_j = k, m) = (1 - c_{jk}) \frac{\exp\left[Da^*_{jk}(\theta - b^*_{jk})\right]}{1 + \exp\left[Da^*_{jk}(\theta - b^*_{jk})\right]}, \qquad (6)$$

where D is a constant equaling 1.7, $a^*_{jk} = a_{jk} - a_{jm}$, $b^*_{jk} = b_{jk} - b_{jm}$, and $c^*_{jk} = c_{jk} - c_{jm}$. Equation 4 specifies the kth contrast function of the DM. There will be m – 1 contrast functions, one for each of the m -1 distractors. The constant D is included so that the contrast functions underlying the DM are similar in form to the three-parameter logistic model, thus allowing the parameters of the DM contrast functions to be interpreted in a similar fashion as those of widely used dichotomous response models. The nature of the contrast function in Equation 6 is similar to the NRM, with the addition of the $(1 - c_{jk})$ term. As a result, the DM is founded on a theory and parameterization that is consistent with the NRM as opposed to the DLT, which is founded on the distractor selection ratio of Equation 4.

The constraints placed on the parameters of the DM are as follows: $a_{jm} = b_{jm} = c_{jm} = 0$. Hence, Equation 6 can be written as

$$P(Y_j = k \mid \theta, Y_j = k, m) = (1 - c_{jk}) \frac{\exp\left[Da_{jk}(\theta - b_{jk})\right]}{1 + \exp\left[Da_{jk}(\theta - b_{jk})\right]} \ . \qquad (7)$$

The discrimination of option k, a_{jk}, is assumed to be negative in value causing the kth contrast function to be monotonically decreasing, thus indicating that the probability of selecting the kth distractor (given the selection of either the correct response of the kth distractor) decreases with θ. The steepness of this decrease is determined by the a_{jk} parameter, the location of the function is determined by the b_{jk} parameter, and the upper asymptote of this function is equal to $1\text{-}c_{jk}$. The kth contrast function described in Equation 7 has a resemblance to the three-parameter logistic IRT model (Lord, 1980) commonly employed for dichotomously scored items. The distinction between Equation 7 and the three-parameter logistic model resides in the use of the c-parameter; in Equation 7, $1\text{-}c_{jk}$ corresponds to the upper asymptote of the kth contrast function, whereas the c-parameter of the three-parameter logistic model corresponds to the lower asymptote of correct response.

The unique property of the DM resides in how we conceptualize the contrast function for an individual with an arbitrarily low value of ability. If we can assume that the response of this individual (with an arbitrarily low value of θ) was the result of some level of guessing, then the probability of selecting the kth distractor given the selection of either the correct option or the kth distractor is expected to be less than unity. That is, an individual with a very low level of θ will not have a zero probability of selecting the correct response – the correct response can be attained through chance (i.e. guessing). In the case of complete random guessing, the probability of selecting the correct option given the choice between the correct option and the kth distractor would be on the order of .5 (a probability of .5 of selecting the correct option and a probability of .5 of selecting the kth distractor). A distractor that had particularly attractive features for individuals with low levels of θ (in relation to the correct option), or that was particularly misleading, could have a contrast function with $c_{jk} < .5$. This is where the DM differs from the NRM – the NRM assumes that $P(Y_j = m|\theta, Y_j = k, m)$ approaches zero as θ becomes arbitrarily low, while the DM assumes that $P(Y_j = m|\theta, Y_j = k, m)$ approaches c_{jk} as θ becomes arbitrarily low. Indeed, the NRM is equal to the DM for which $c_{jk} = 0$ for all k, a relationship that is described in greater detail below.

Figure 1 illustrates the contrast functions of the DM associated with a hypothetical four-option multiple-choice item. Because there are four options, there are three contrast functions, labeled CF-1, CF-2, and CF-3. This item has parameters $a_1 = -1.5$, $a_2 = -1$, $a_3 = -1$, $b_1 = -1$, $b_2 = 0$, $b_3 = 1$, $c_1 = 0.2$, $c_2 = 0.5$, and $c_3 = 0.3$. Based on these parameters, we see that CF-1 has the largest a-parameter, indicating that the distinction between the correct option and distractor 1 holds the greatest discriminating power between low and high levels of θ. The b-parameter corresponds to the horizontal location of the contrast function; CF-3 has the largest value of the b-parameter and thus lies furthest to the right. CF-2 has the largest c-parameter ($c_2 = 0.5$), and thus the lowest upper asymptote. As a result, when faced with a decision between the correct option and distractor 2, respondents with very low levels of θ are responding at random. In contrast, CF-1 has a c-parameter value of 0.2, suggesting that distractor 2 is particularly attractive (or deceiving) for respondents with very low levels of θ.

Based on the contrast function defined in Equation 7, the DM defines the conditional probability of selecting the kth response option using

$$P(Y_j = k \mid \theta) = \frac{w_{jk}}{\sum_{k=1}^{m} w_{jk}} \tag{8}$$

where

$$w_{jk} = \frac{(1 - c_{jk})\exp\left[Da_{jk}(\theta - b_{jk})\right]}{1 + c_{jk}\exp\left[Da_{jk}(\theta - b_{jk})\right]}. \tag{9}$$

The constraints placed on the parameters of the DM are as follows: $a_{jm} = b_{jm} = c_{jm} = 0$. The DM has $3(m - 1)$ free parameters, which is more than that of the NRM, equal to that of the DLT, and fewer than that of the Thissen and Steinberg's (1984) MCM. It is also relevant to note that when $c_{jk} = 0$ for $k = 1, 2, \ldots, m - 1$, w_{jk} reduces to

$$w_{jk} = \exp\left[Da_{jk}(\theta - b_{jk})\right] , \tag{10}$$

which leads the DM to be equivalent to a rescaled version of the NRM in which the b^*_{jk} parameter of the NRM is equivalent to $Da_{jk}b_{jk}$ in Equation 10 and the a^*_{jk} parameter of the NRM is equivalent to Da_{jk} in Equation 10. That is, under the condition of $c_{jk} = 0$ for all distractors the DM holds a form that is algebraically equivalent to the NRM.

A pivotal property of the DM is its relationship to the NRM. Examining the contrast functions of the DM (Equation 7) and the NRM (Equation 2) we see that that they differ by the factor $1 - c_{jk}$. This is analogous to the distinction between the two- and three-parameter logistic dichotomous response models. As such, the DM can be conceptualized as the three-parameter logistic analog of the NRM. That is, the distinction between the DM and the NRM is analogous to the distinction between the three-parameter and two-parameter logistic dichotomous response models.

The derivation of the DM is founded on the formulation of the contrast function defined in Equation 7. Because the kth contrast function represents $P(Y_j = k|Y_j = k, m)$, which is equal to $P(Y_j = k|\theta)/[P(Y_j = k|\theta) + P(Y_j = m|\theta)]$, it follows that

$$\frac{P(Y_j = k \mid \theta)}{P(Y_j = k \mid \theta) + P(Y_j = m \mid \theta)} = (1 - c_{jk}) \frac{\exp[Da_{jk}(\theta - b_{jk})]}{1 + \exp[Da_{jk}(\theta - b_{jk})]}. \tag{11}$$

Isolating $P(Y_j = k|\theta)$ on the left side of the equation yields the following form for $P(Y_j = k \mid \theta)$

$$P(Y_j = k) = w_{jk} P(Y_j = m), \tag{12}$$

where

$$w_{jk} = \frac{(1 - c_{jk})\exp[Da_{jk}(\theta - b_{jk})]}{1 + c_{jk}\exp[Da_{jk}(\theta - b_{jk})]}. \tag{13}$$

It must be the case that, conditional on θ, the sum of the probability of the correct response and each of the $m - 1$ distractors is equal to unity. As a result, the probability of correct response can be expressed as

$$P(Y_j = m \mid \theta) = 1 - \sum_{k=1}^{m-1} P(Y_j = k)$$

$$= 1 - P(Y_j = m \mid \theta) \sum_{k=1}^{m-1} w_{jk} \ . \quad (14)$$

Isolating the term for $P(Y_j = m|\theta)$ on the left side yields the following

$$P(Y_j = m \mid \theta) = \frac{1}{1 + \sum_{k=1}^{m-1} w_{jk}} \ . \quad (15)$$

Substituting the result of Equation 15 into Equation 12 yields the form for $P(Y_j = k|\theta)$, given by

$$P(Y_j = k \mid \theta) = \frac{w_{jk}}{1 + \sum_{k'=1}^{m-1} w_{jk'}} \ . \quad (16)$$

Assuming that the constraint of $a_{jm} = b_{jm} = c_{jm} = 0$ has been imposed, the form of the DM shown in Equation 8 can be used to specify the conditional probability of the kth response option, for $k = 1, 2, \ldots, m$.

Although not immediately apparent from Equation 16 and the previous equations presenting the derivation of the DM, the DM can be shown to be a reparameterization of the DLT. In particular, the Revuelta's (2005) presentation of the DLT asserts that (see Equation 4)

$$\psi_k(\theta) = \frac{P(Y_j = m \mid \theta)}{P(Y_j = k \mid \theta)} = \frac{\pi_{jm}}{\pi_{jk}} \left[1 + \exp(b_{jk} + a_{jk}\theta)\right].$$

Under the DM, $\psi_k(\theta)$ can be expressed by

$$\psi_k(\theta) = w_k^{-1} = \frac{1 + c_{jk} \exp\left[Da_{jk}(\theta - b_{jk})\right]}{(1 - c_{jk}) \exp\left[Da_{jk}(\theta - b_{jk})\right]}$$

$$= \frac{c_{jk} + \exp[-Da_{jk}(\theta - b_{jk})]}{1 - c_{jk}}$$

$$= \frac{c_{jk}}{1 - c_{jk}}\left(1 + c_{jk}^{-1}\exp[-Da_{jk}(\theta - b_{jk})]\right). \quad (17)$$

Letting $v_{jk} = b_{jk} + \log(c_{jk})(Da_j)^{-1}$, ψ_k under the DM can be expressed as

$$\psi_k(\theta) = \frac{c_{jk}}{1 - c_{jk}}\left(1 + c_{jk}^{-1}\exp[-Da_{jk}(\theta - v_{jk})]\right)$$

$$= \frac{c_{jk}}{1 - c_{jk}}\left(1 + c_{jk}^{-1}\exp[-Da_{jk}(\theta - b_{jk})]\exp[Da_{jk}\log(c_{jk})(Da_j)^{-1}]\right)$$

$$= \frac{c_{jk}}{1 - c_{jk}}\left(1 + c_{jk}^{-1}\exp[-Da_{jk}(\theta - b_{jk})]c_{jk}\right)$$

$$= \frac{c_{jk}}{1 - c_{jk}}\left(1 + \exp[-Da_{jk}\theta + Da_{jk}b_{jk}]\right), \quad (18)$$

which is equivalent in form to $\psi_k(\theta)$ under the DLT (see Equation 4). As a result, the DM can be expressed as the DLT when the following transformations are imposed on the DM parameterization

$$\frac{\pi_{jm}}{\pi_{jk}} = \frac{c_{jk}}{1 - c_{jk}}$$

$$\alpha_{jk} = -Da_{jk}$$

$$\beta_{jk} = Da_{jk}b_{jk} - \log(c_{jk}). \quad (19)$$

An appealing property of the DM is the interpretability of the guessing parameters (c_{jk}) with respect to the contrast functions. A value $c_{jk} = 0.5$ for all k corresponds to the situation of item responses being determined purely by random guessing for individuals with very low levels of θ. In this situation $P(Y_j = k|\theta) \leq P(Y_j = m|\theta)$ for all k, regardless of

θ. A value of $c_{jk} < 0.5$ for the kth distractor provides evidence that the distractor is an attractive option relative to the correct response for individuals with arbitrarily low levels of θ. Similarly, a value of $c_{jk} > 0.5$ for the kth distractor provides evidence that the distractor is not an attractive option relative to the correct response for individuals with arbitrarily low levels of θ, in which case the correct option may contain information or properties that is making it an appealing choice for individuals with very low levels of θ.

An additional appealing property of the DM corresponds to the form of the probability of selecting the correct response, given the choice between the correct response and the kth distractor. This form is given by

$$
\begin{aligned}
P(Y_j = m \mid \theta, Y_j = k, m) &= 1 - P(Y_j = k \mid \theta, Y_j = k, m) \\
&= 1 - (1 - c_{jk}) \frac{\exp[Da_{jk}(\theta - b_{jk})]}{1 + \exp[Da_{jk}(\theta - b_{jk})]} \\
&= \frac{1 + c_{jk} \exp[Da_{jk}(\theta - b_{jk})]}{1 + \exp[Da_{jk}(\theta - b_{jk})]} \\
&= \frac{1 - c_{jk} + c_{jk}}{1 + \exp[Da_{jk}(\theta - b_{jk})]} + \frac{c_{jk} \exp[Da_{jk}(\theta - b_{jk})]}{1 + \exp[Da_{jk}(\theta - b_{jk})]} \\
&= \frac{1 - c_{jk}}{1 + \exp[Da_{jk}(\theta - b_{jk})]} + c_{jk} \frac{1 + \exp[Da_{jk}(\theta - b_{jk})]}{1 + \exp[Da_{jk}(\theta - b_{jk})]} \\
&= c_{jk} + \frac{1 - c_{jk}}{1 + \exp[Da_{jk}(\theta - b_{jk})]} \\
&= c_{jk} + (1 - c_{jk}) \frac{\exp[Da'_{jk}(\theta - b_{jk})]}{1 + \exp[Da'_{jk}(\theta - b_{jk})]}, \qquad (20)
\end{aligned}
$$

where $a'_{jk} = -a_{jk}$. The form shown in Equation 20 is the familiar three-parameter logistic model commonly employed for dichotomously scored items. As a result, under the DM the probability of correct response given the choice between the correct option or the *k*th distractor follows the three-parameter logistic model commonly employed for dichotomously scored items.

Figure 2 presents the option characteristic curves (OCCs) of the DM associated with a hypothetical four-option multiple-choice item for which the first three options correspond to distractors and the fourth option is the correct option. The DM parameters for this item are $a_1 = -1.5$, $a_2 = -1$, $a_3 = -1$, $b_1 = -1$, $b_2 = 0$, $b_3 = 1$, $c_1 = 0.2$, $c_2 = 0.5$, and $c_3 = 0.3$. Note that this is the same hypothetical item for which the contrast functions are displayed in Figure 1. The shape of the OCCs indicates that distractor 3 is particularly attractive for respondents having moderate levels of θ and distractor 1 is particularly attractive for respondents having low levels of θ. The ordering of these two distractors (i.e., distractor 3 is attractive for respondents at higher levels of θ than is distractor 1) is attributable to the larger value of the *b*-parameter for distractor 3 ($b_3 = 1$) than for distractor 1 ($b_1 = -1$). Distractor 2 is relatively unattractive for respondents of all levels of θ, which is attributable to its relatively low value of the *a*-parameter ($a_2 = -1$) and the high-value of the *c*-parameter ($c_2 = 0.5$). For very low values of θ distractor 1 is a highly attractive option, which is attributable to its relatively low value of *c* ($c_1 = 0.2$). In contrast, distractor 2 has a low probability of being selected by individuals with a very low value of θ, due to its high value of *c* ($c_2 = 0.5$).

Parameter Estimation for the Distractor Model

As in most IRT models, estimates of the DM parameters can be obtained using marginalized maximum likelihood estimation (MMLE; Bock & Aitkin, 1981). Alternatively, as Patz and Junker (1999a, 1999b) have shown, IRT model parameters can also be estimated using Markov chain Monte Carlo (MCMC) methods. Although computationally more intensive, the latter approach is easier to implement primarily because it does not require the first and second order derivatives to arrive at the solution.

For estimation purposes, the model is parameterized as follows:

$$\theta_i \sim N(0,1),$$

$$a_{jk} \sim \mathrm{U}(0.5, 2.0),$$

$$b_{jk} \sim \mathrm{U}(-3.0, 3.0),$$

$$c_{jk} \sim \mathrm{U}(0.0, 0.5),$$

$$Y_{jk} \mid \theta_i, a_{jk}, b_{jk}, c_{jk} \sim P(Y_j = k \mid \theta_i),$$

where the subscript *i* refers to the *i*th respondent. The joint posterior distribution of the parameters is

$$P(\boldsymbol{\theta}, \boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c} \mid \boldsymbol{Y}) \propto L(\boldsymbol{Y} \mid \boldsymbol{\theta}, \boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}) P(\boldsymbol{\theta}) P(\boldsymbol{a}) P(\boldsymbol{b}) P(\boldsymbol{c}),$$

where $L(\boldsymbol{Y} \mid \boldsymbol{\theta}, \boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}) = \prod_{i,j,k} P(Y_j = k \mid \theta_j)^{Y_{ijk}}$, $P(\boldsymbol{\theta}) = \prod_i P(\theta_i)$, $P(\boldsymbol{a}) = \prod_{j,k} P(a_{jk})$, $P(\boldsymbol{b}) = \prod_{j,k} P(b_{jk})$, $P(\boldsymbol{c}) = \prod_{j,k} P(c_{jk})$, and $Y_{ijk} = 1$ if and only if $Y_{ij} = k$. The full conditional distributions of $\boldsymbol{\theta}$, $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ are not known distributions so sampling from these distributions requires the use of the Metropolis-Hastings within Gibbs algorithm (Casella & George, 1995; Chib & Greenberg, 1995; Gamerman, 1997; Gelman, Carlin, Stern, & Rubin, 2003; Gilks, Richardson, & Spiegelhalter, 1996; Tierney, 1994). The means and standard deviations of the draws, averaged across the chains, serve as the estimated parameters and standard errors, respectively.

Model Fit

The assessment of model fit for polytomous models is a difficult endeavor in general (Drasgow, Levine, Tsien, Williams, & Mead, 1995; Roberts, Donoghue, & Laughlin, 2000; Thissen & Steinberg, 1997), and this difficulty applies equally to the DM. Chi-square tests of fit become impractical with more than just a few items due to the large number of possible response patterns (Bock, 1997; Thissen & Steinberg, 1997). Chi-square tests have also been proposed based on creating dyads or triads of items (Drasgow et al., 1995), but even these approaches are difficult to implement without large sample sizes. In addition, conventional measures of fit based on the mean squared error between the observed and expected response are not applicable due to the nominal nature of the response variable.

One approach for examining fit of polytomous models applied to multiple-choice items is to consider diagnostic plots that compare the observed and expected proportion selecting each response option within a finite number of points along the latent trait continuum (Drasgow et al., 1995). For example, one could segment the latent continuum into 12 strata (e.g., -3.00 to -2.50, -2.49 to -2.00, …, 2.50 to 3.00) and within each stratum compare the observed proportion selecting each response option to the proportions expected under the DM evaluated at the midpoint of the stratum. The observed and expected proportion plots can be examined visually to identify particularly misfitting items. In addition, the root mean squared error (RMSE) can be computed for each option by considering the difference between the observed and expected proportions within each stratum (i.e., computing the mean of the squared differences across the strata, and then taking the square root of the obtained mean). The value of the RMSE for each option can be used to identify options that are experiencing substantial lack of fit. The RMSE also can be computed across all options to yield an item-level index of fit. We stress, however, that the resulting RMSE values are not to be confused with a hypothesis test of fit, and should be used only as a guide in the examination of fit.

An Illustrative Example

We illustrate the application of the DM using the responses of a random sample of 2500 students to 20 multiple-choice items of the algebra scale of a mathematics placement test of a large Midwestern university (UWCTP, 2006). The entire test contained 85 items covering mathematics basics, algebra, and trigonometry. This illustration is based on only the first 20 items of the algebra scale. Each item in this illustration contained five response options (correct option and four distractors). We used only the first 20 items for illustrative purposes.

The estimates of the DM parameters, including the ability estimates, were obtained using the MCMC algorithm described previously. Four independent chains were started at random, and each chain was run for a total of 50000 iterations. A burn-in of 10000 iterations was used, and only every tenth draw was saved. Using the multivariate potential scale reduction factor (Brooks & Gelman, 1998), the convergence statistic was computed to be 1.15, indicating that the chains had converged. The means and standard deviations of the draws, averaged across the chains, were used as the estimates of the parameters and the standard errors, respectively. The code for this algorithm was written in Ox (Doornik, 2003), and can be made available by contacting the authors.

The estimated item parameters, and their associated standard error estimates, are presented in Table 1. Options 1 to 4 correspond to distractors and option 5 corresponds to the correct response. Of particular interest are the values of the *c*-parameter estimates. Across all 20 items, the *c*-parameter estimates ranged between 0.13 (c_4 of Item 14) and 0.49 (c_4 of Item 17). Typically, the *c*-parameter estimates ranged between 0.20 and 0.45, with the mean and standard deviation across all 20 items of 0.32 and 0.08, respectively. The mean and standard deviation of each of the *a*-, *b*-, and *c*-parameters taken across all 20 items are displayed in Table 2.

To gain a better understanding of the relationship between the estimated item parameters and the form of the DM, the OCCs were plotted for several of the test items. Figure 3 displays the OCCs for Item 6. Of particular interest for Item 6 is distractor 3, which displayed a relatively high probability of selection for the θ range of -1 to -3. The relatively high probability associated with distractor 3 in the θ range of -1 to -3 is attributable to the relatively high discrimination of the third contrast function ($a_3 = -1.18$) coupled with the location of the third contrast function ($b_3 = -0.79$) that sits far to the right of the other contrast functions. In addition, the relatively low value of the *c*-parameter for the third contrast function ($c_3 = 0.22$) leads the probability of selecting the third distractor to be above that of the other options at the lower levels of θ.

Figure 4 displays the OCCs for Item 11. For this item, distractor 1 had a relatively high probability of selection in the θ range of -2 to -1, which is attributable to the relatively high *a*- and *b*-parameters of the first contrast function ($a_1 = -1.12$ and $b_1 = -0.33$). Also of interest is that distractor 3 reached a relatively high probability of selection at the very low levels of θ, which is attributable to the relatively low value of the c-parameter ($c_3 = 0.20$).

Figure 5 displays the OCCs for Item 19, a relatively difficult item for which the OCCs of the four distractors were determined largely by the c-parameters of their respective contrast functions. In particular, for $\theta < 0$, the ordering of the probability of selection for the four distractors is inversely related to the c-parameter of the associated contrast function: distractor 2 has the highest probability of selection and $c_2 = 0.19$, distractor 3 has the second highest probability of selection and $c_3 = 0.22$, distractor 1 has the third highest probability of selection and $c_1 = 0.27$, and distractor 4 has the lowest probability of selection (of the distractors) and $c_4 = 0.41$. This item clearly illustrates the utility of the c-parameter in the DM, and how the c-parameter is inversely related to the probability of selection as θ becomes arbitrarily low.

Fit was examined by comparing the observed and expected proportion selecting each response option within ten intervals along the latent continuum. The latent continuum was placed on a standard metric (one logit equaled one unit on the latent metric), and the ten intervals were 0.5 logits in length beginning at -2.5 and extending to 2.5 (i.e., -2.50 to -2.01, -2.00 to -1.51, …., 2.00 to 2.50). At the midpoint of each of the ten intervals the expected proportion selecting each response option for the item in question was computed, and compared to the observed proportions for the respondents in the respective interval. The mean squared error for each response option was computed by summing the squared differences between the expected and observed proportions, and the RMSE for each response option was obtained by taking the square root of the mean squared error. To obtain an index of fit across all response options, an aggregated RMSE value was computed by considering the summated squared differences across all five response options.

Upon examination of the data, only six observations were located in the lowest interval (-2.5 to -2.01), and thus this interval was excluded from the computation of the RMSE values. Table 2 displays the resulting values of RMSE for each response option, and aggregated across all response options. The RMSE values were typically less than 0.05 for each distractor, providing evidence of relatively good fit. The values of RMSE for the correct option tended to be slightly larger than those of the distractors, which is likely attributable to the larger scale of the correct response; the correct response spans probabilities from near zero to near unity, while the distractors typically span probabilities from near zero to less than .5. The values of the aggregated RMSE taken across all response options ranged between 0.018 and 0.043, which provides further evidence of relatively good fit of the model to the data.

In addition to the RMSE values shown in Table 2, fit plots (plots of the expected and observed proportion selecting each response option within each stratum) were constructed for each item. The fit plots were visually inspected to identify items displaying substantial lack of fit. Upon visual inspection of the fit plots, no items were flagged as having substantial lack of fit.

Discussion

In this paper we proposed the distractor model (DM), an adaptation of Revuelta's (2005) DLT response model for multiple-choice items. The advantageous features of the DM include: (a) the DM uses a parameterization that is consistent with that of the NRM; (b) the DM uses fewer parameters than Thissen and Steinberg's (1984) multiple-choice response model; (c) the DM is based on contrast functions that are parameterized according to a three-parameter logistic model that is similar to that commonly employed for dichotomously scored items, and thus is based on a theory and form that is pervasive in the IRT literature. The DM was applied to a 20-item test of multiple-choice items, and the fit of the data to the DM was deemed good through visual inspection of fit plots. These results suggest that the DM holds promise to being a viable alternative for modeling multiple-choice item responses.

Adding the DM to the family of polytomous models appropriate for multiple-choice item formats can serve several advantageous functions. First, the DM is based on different model assumptions than the NRM and the MCM. In particular, the DM asserts that each contrast function follows a three-parameter logistic form, which distinguishes it from the NRM and the MCM. As a result, the DM offers a novel way to conceptualize, model, and interpret responses to multiple-choice items. Second, because the assumptions underlying the DM are different than those of the NRM and MCM, it offers an alternative model to simulate responses to multiple-choice items. Being able to simulate data using a variety of plausible models is a useful property of simulation studies evaluating models for multiple-choice items.

While the DM incorporates a theory and parameterization for guessing in multiple-choice items, additional research is required to evaluate whether the DM provides better fit to multiple-choice item responses than other polytomous models such as the NRM and the MCM. Previous research (Drasgow et al., 1995) found modest differences in the fit of NRM and the MCM to multiple-choice items, and additional research is required to evaluate whether the fit of the DM to real data is comparable to that of the MCM and NRM. In addition, a comparison of the stability of the DM parameter estimates to those of the MCM and NRM would be important in shedding light on which model holds the greatest value across a range a conditions.

Appendix

$$P(Y_j = k \mid \theta, Y_j = k, m) = \frac{P(Y_j = k \mid \theta)}{P(Y_j = k \mid \theta) + P(Y_j = m \mid \theta)}$$

$$= \frac{\exp(b_{jk} + a_{jk}\theta) \Big/ \sum_{k'=1}^{m} \exp(b_{jk'} + a_{jk'}\theta)}{\exp(b_{jk} + a_{jk}\theta) \Big/ \sum_{k'=1}^{m} \exp(b_{jk'} + a_{jk'}\theta) + \exp(b_{jm} + a_{jm}\theta) \Big/ \sum_{k'=1}^{m} \exp(b_{jk'} + a_{jk'}\theta)}$$

$$= \frac{\exp(b_{jk} + a_{jk}\theta)}{\exp(b_{jk} + a_{jk}\theta) + \exp(b_{jm} + a_{jm}\theta)}$$

$$= \frac{\exp(b_{jk} + a_{jk}\theta) / \exp(b_{jm} + a_{jm}\theta)}{1 + \exp(b_{jk} + a_{jk}\theta) / \exp(b_{jm} + a_{jm}\theta)}$$

$$= \frac{\exp\left[(b_{jk} - b_{jm}) + \theta(a_{jk} - a_{jm})\right]}{1 + \exp\left[(b_{jk} - b_{jm}) + \theta(a_{jk} - a_{jm})\right]}$$

$$= \frac{\exp(b^*_{jk} + a^*_{jk}\theta)}{1 + \exp(b^*_{jk} + a^*_{jk}\theta)}$$

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Author Note

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Table 1

Item Parameter Estimates and Estimated Standard Errors

Item	a_1	a_2	a_3	a_4	b_1	b_2	b_3	b_4	c_1	c_2	c_3	c_4
1	-0.57	-0.81	-1.14	-0.52	-2.57	-1.90	-0.35	-0.75	0.32	0.26	0.46	0.47
	(0.05)	(0.10)	(0.15)	(0.02)	(0.27)	(0.30)	(0.14)	(0.14)	(0.12)	(0.14)	(0.04)	(0.03)
2	-1.78	-1.15	-1.05	-1.37	-0.29	-0.23	-1.39	-0.64	0.35	0.21	0.23	0.44
	(0.15)	(0.13)	(0.13)	(0.20)	(0.08)	(0.11)	(0.24)	(0.15)	(0.05)	(0.06)	(0.13)	(0.06)
3	-0.57	-1.02	-1.01	-0.83	-0.33	-0.98	-0.50	-1.38	0.40	0.40	0.43	0.37
	(0.06)	(0.14)	(0.14)	(0.12)	(0.29)	(0.22)	(0.20)	(0.32)	(0.08)	(0.09)	(0.07)	(0.12)
4	-1.48	-1.53	-1.63	-0.66	-0.06	-0.22	-0.11	0.35	0.45	0.29	0.32	0.31
	(0.21)	(0.18)	(0.19)	(0.11)	(0.11)	(0.10)	(0.10)	(0.24)	(0.04)	(0.05)	(0.05)	(0.07)
5	-0.84	-0.98	-0.78	-0.65	-0.16	-0.19	-0.33	0.12	0.24	0.30	0.40	0.41
	(0.14)	(0.15)	(0.13)	(0.10)	(0.23)	(0.20)	(0.29)	(0.27)	(0.09)	(0.08)	(0.09)	(0.07)
6	-1.47	-1.06	-1.18	-0.81	-2.06	-1.36	-0.79	-2.10	0.29	0.29	0.22	0.24
	(0.24)	(0.13)	(0.13)	(0.11)	(0.25)	(0.22)	(0.16)	(0.31)	(0.14)	(0.13)	(0.09)	(0.14)
7	-0.94	-1.47	-1.21	-0.72	-0.46	-0.42	-1.07	-0.17	0.36	0.27	0.30	0.18
	(0.16)	(0.19)	(0.17)	(0.09)	(0.31)	(0.12)	(0.21)	(0.21)	(0.12)	(0.07)	(0.12)	(0.08)
8	-0.80	-1.16	-0.80	-0.80	-0.61	0.20	-0.83	0.15	0.18	0.24	0.33	0.46
	(0.11)	(0.15)	(0.13)	(0.13)	(0.24)	(0.10)	(0.33)	(0.20)	(0.10)	(0.04)	(0.12)	(0.05)
9	-1.19	-0.65	-0.77	-0.87	-0.49	-1.69	-1.28	-0.51	0.35	0.24	0.38	0.43
	(0.18)	(0.08)	(0.10)	(0.12)	(0.20)	(0.36)	(0.30)	(0.23)	(0.09)	(0.15)	(0.11)	(0.07)
10	-1.30	-1.05	-1.78	-1.10	-0.90	-1.05	-0.83	-1.39	0.38	0.29	0.44	0.36
	(0.18)	(0.13)	(0.15)	(0.15)	(0.19)	(0.23)	(0.09)	(0.25)	(0.10)	(0.12)	(0.05)	(0.12)
11	-1.12	-0.70	-0.71	-1.14	-0.33	-2.14	-0.91	-0.96	0.23	0.26	0.20	0.44
	(0.13)	(0.09)	(0.09)	(0.16)	(0.12)	(0.33)	(0.29)	(0.20)	(0.06)	(0.14)	(0.12)	(0.07)
12	-0.58	-0.91	-0.85	-1.13	-2.26	-1.55	-1.06	-0.73	0.26	0.28	0.37	0.42
	(0.05)	(0.11)	(0.11)	(0.13)	(0.32)	(0.27)	(0.28)	(0.16)	(0.14)	(0.13)	(0.11)	(0.07)
13	-0.70	-0.87	-1.16	-0.66	-1.67	-1.13	-0.55	-2.11	0.29	0.34	0.45	0.32
	(0.08)	(0.12)	(0.15)	(0.09)	(0.33)	(0.30)	(0.14)	(0.37)	(0.14)	(0.13)	(0.05)	(0.14)
14	-1.41	-1.81	-1.35	-0.98	-0.60	-0.52	-0.55	-0.35	0.31	0.31	0.36	0.13
	(0.20)	(0.13)	(0.20)	(0.10)	(0.17)	(0.09)	(0.18)	(0.12)	(0.10)	(0.06)	(0.09)	(0.06)
15	-1.22	-0.55	-1.11	-0.60	0.75	0.32	1.07	-0.21	0.28	0.44	0.46	0.45
	(0.15)	(0.04)	(0.25)	(0.08)	(0.08)	(0.25)	(0.14)	(0.24)	(0.02)	(0.05)	(0.03)	(0.05)
16	-0.86	-0.67	-0.77	-0.65	0.52	-1.75	-2.29	-1.53	0.29	0.31	0.31	0.32
	(0.15)	(0.09)	(0.12)	(0.08)	(0.16)	(0.38)	(0.35)	(0.35)	(0.06)	(0.14)	(0.13)	(0.13)
17	-0.52	-0.69	-0.84	-0.51	-2.22	-1.49	-1.07	0.11	0.38	0.24	0.21	0.49
	(0.02)	(0.08)	(0.10)	(0.01)	(0.27)	(0.33)	(0.24)	(0.13)	(0.10)	(0.14)	(0.11)	(0.01)
18	-0.87	-0.75	-0.85	-0.73	-0.90	-0.54	-0.32	-2.47	0.37	0.24	0.33	0.27
	(0.12)	(0.11)	(0.14)	(0.10)	(0.27)	(0.28)	(0.27)	(0.31)	(0.11)	(0.11)	(0.10)	(0.14)
19	-1.17	-1.81	-1.38	-1.36	0.07	0.41	0.59	0.11	0.27	0.19	0.22	0.41
	(0.17)	(0.13)	(0.19)	(0.23)	(0.14)	(0.05)	(0.08)	(0.15)	(0.06)	(0.02)	(0.03)	(0.06)
20	-0.69	-0.76	-1.00	-1.02	-0.26	-1.30	-0.20	-0.70	0.23	0.25	0.31	0.41
	(0.11)	(0.10)	(0.14)	(0.14)	(0.28)	(0.31)	(0.18)	(0.21)	(0.10)	(0.14)	(0.07)	(0.08)

Note. Values in parentheses correspond to the estimated standard error of the parameter estimate.

Table 2
Mean and Standard Deviation of the Item Parameter Estimates

	a_1	a_2	a_3	a_4	b_1	b_2	b_3	b_4	c_1	c_2	c_3	c_4
Mean	1.00	1.02	1.07	0.85	-0.74	-0.88	-0.64	-0.76	0.31	0.28	0.34	0.37
SD	0.36	0.37	0.30	0.26	0.94	0.77	0.71	0.83	0.07	0.06	0.09	0.10

Table 3
RMSE Values for Each Response Option

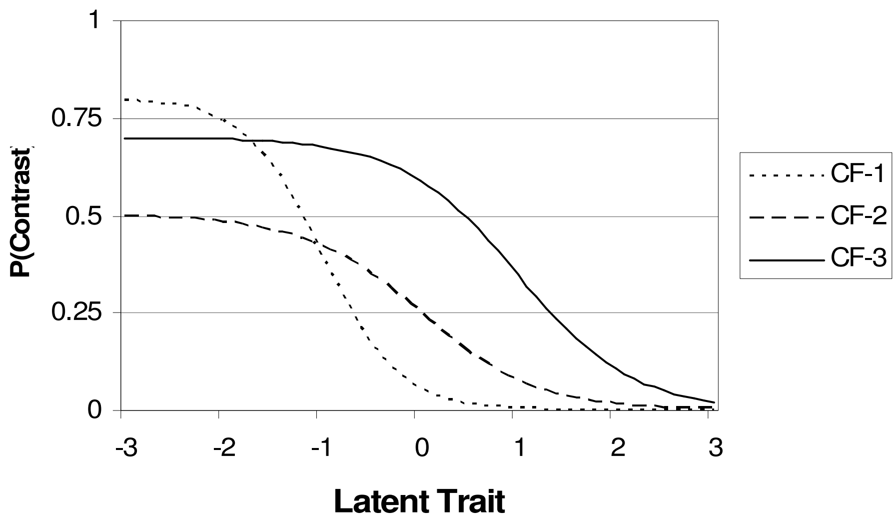
	Response Option					
Item	1	2	3	4	5	Aggregated
1	0.032	0.038	0.010	0.047	0.045	0.037
2	0.015	0.031	0.047	0.009	0.022	0.028
3	0.027	0.010	0.028	0.015	0.030	0.024
4	0.013	0.022	0.014	0.027	0.033	0.023
5	0.027	0.018	0.016	0.023	0.047	0.029
6	0.025	0.019	0.025	0.016	0.050	0.030
7	0.015	0.012	0.023	0.031	0.042	0.027
8	0.028	0.032	0.017	0.026	0.033	0.028
9	0.012	0.016	0.013	0.015	0.042	0.022
10	0.011	0.019	0.009	0.005	0.033	0.018
11	0.021	0.011	0.036	0.019	0.047	0.030
12	0.020	0.027	0.011	0.031	0.027	0.024
13	0.015	0.015	0.016	0.013	0.041	0.023
14	0.043	0.038	0.018	0.039	0.034	0.035
15	0.029	0.050	0.027	0.021	0.068	0.043
16	0.034	0.007	0.019	0.033	0.050	0.032
17	0.016	0.017	0.036	0.053	0.068	0.043
18	0.015	0.028	0.023	0.010	0.041	0.026
19	0.016	0.020	0.034	0.012	0.035	0.025
20	0.056	0.014	0.019	0.012	0.051	0.036

Note. Response options 1, 2, 3, and 4 correspond to distractors. Option 5 is the correct response. The aggregated column corresponds to the RMSE aggregated across all five response options.

Figure 1

Contrast Functions for a Hypothetical Four-Option Multiple-Choice Item Parameterized

According to the DM

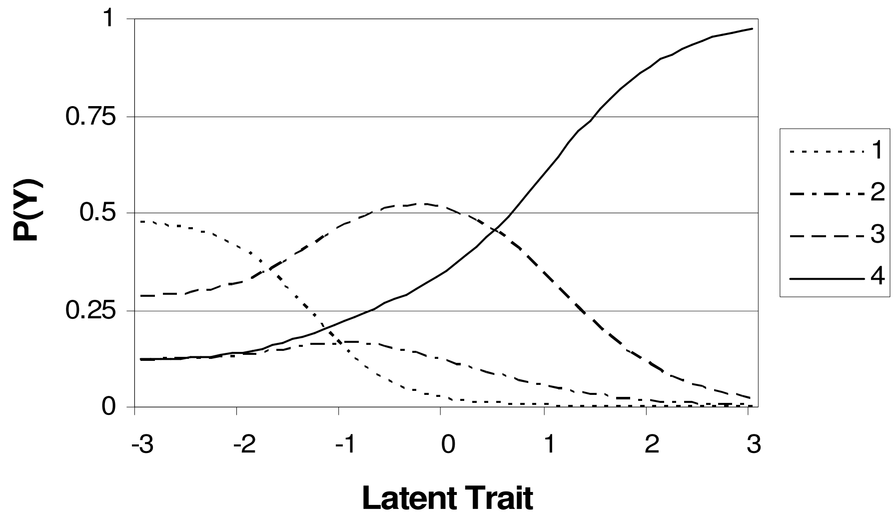


Note. The three contrast functions are labeled CF-1 (contrasts distractor 1 and the correct option), CF-2 (contrasts distractor 2 and the correct option), and CF-3 (contrasts distractor 3 and the correct option). The three contrast functions have parameters: $a_1 = -1.5$, $a_2 = -1$, $a_3 = -1$, $b_1 = -1$, $b_2 = 0$, $b_3 = 1$, $c_1 = 0.2$, $c_2 = 0.5$, $c_3 = 0.3$.

Figure 2

Option Characteristic Curves for a Hypothetical Four-Option Multiple-Choice Item

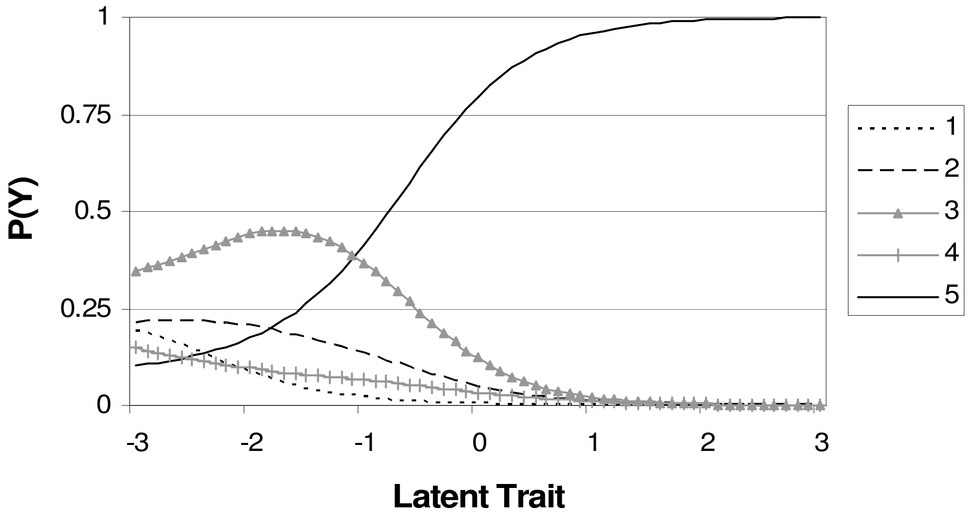
Parameterized According to the DM



Note. The DM parameters are: a_1 = -1.5, a_2 = -1, a_3 = -1, b_1 = -1, b_2 = 0, b_3 = 1, c_1 = 0.2, c_2 = 0.5, c_3 = 0.3. Option 4 corresponds to the correct option.

Figure 3

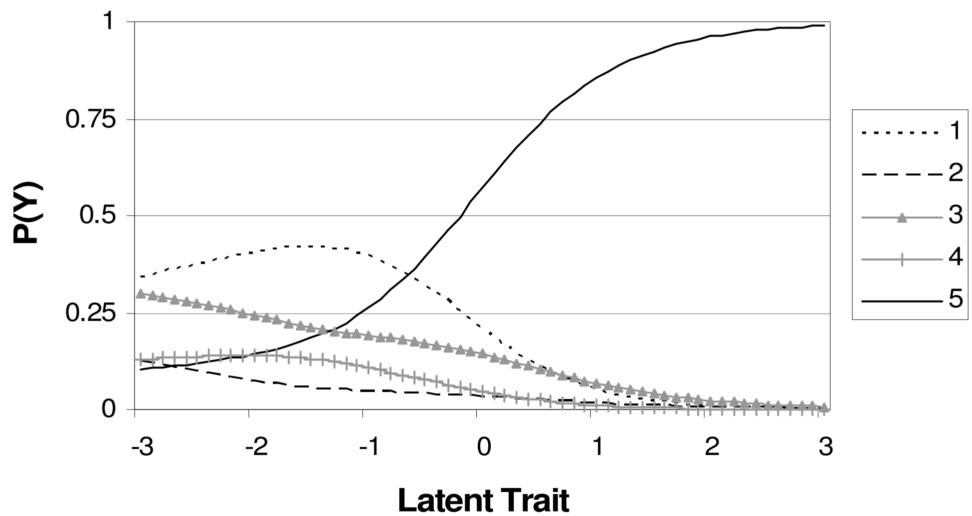
Option Characteristic Curves for Item 6 of the Algebra Test



Note. The DM parameters are: a_1 = -1.47, a_2 = -1.06, a_3 = -1.18, a_4 = -0.81, b_1 = -2.06, b_2 = -1.36, b_3 = -0.79, b_4 = -2.10, c_1 = 0.29, c_2 = 0.29, c_3 = 0.22, c_4 = 0.24. Option 5 corresponds to the correct option.

Figure 4

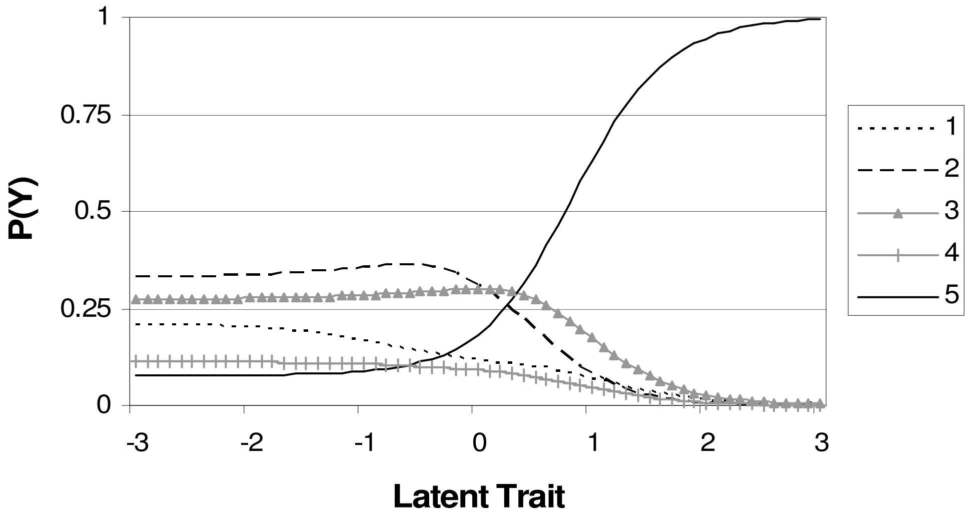
Option Characteristic Curves for Item 11 of the Algebra Test



Note. The DM parameters are: a_1 = -1.12, a_2 = -0.70, a_3 = -0.71, a_4 = -1.14, b_1 = -0.33, b_2 = -2.14, b_3 = -0.91, b_4 = -0.96, c_1 = 0.23, c_2 = 0.26, c_3 = 0.20, c_4 = 0.44. Option 5 corresponds to the correct option.

Figure 5

Option Characteristic Curves for Item 19 of the Algebra Test



Note. The DM parameters are: a_1 = -1.17, a_2 = -1.81, a_3 = -1.38, a_4 = -1.36, b_1 = 0.07, b_2 = 0.41, b_3 = 0.59, b_4 = 0.11, c_1 = 0.27, c_2 = 0.19, c_3 = 0.22, c_4 = 0.41. Option 5 corresponds to the correct option.